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Anisotropic Eddy-Viscosity Concept for Strongly Detached Unsteady Flows

Rémi Bourguet,* Marianna Braza,† Rodolphe Perrin,‡ and Gilles Harran‡
Institut de Mécanique des Fluides de Toulouse, 31400 Toulouse, France

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Nomenclature

a, a_{ij}	=	dimensionless and traceless form of Reynolds stresses
C_i	=	$-a$ and S principal direction misalignment criteria
C_{V_i}	=	$-a$ and S advectable misalignment criteria
C_μ	=	eddy-diffusion coefficient
C_{μ_i}	=	anisotropic eddy-diffusion coefficients
k	=	turbulent kinetic energy, $\overline{u_i u_i}/2$
S, S_{ij}	=	mean strain tensor, $\frac{1}{2}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})$
$(S_i)_{jk}$	=	ith element of S spectral decomposition
U_i	=	mean flow velocity components
$\overline{u_i u_j}$	=	turbulent stresses
v_i^a	=	$-a$ orthogonal normalized eigenvectors
v_i^S	=	S orthogonal normalized eigenvectors
δ_{ij}	=	Kronecker symbol
ε	=	turbulence dissipation rate
η	=	mean flow/turbulent time-scale rate
λ_i^a	=	$-a$ classified by descending order eigenvalues
λ_i^S	=	S classified by descending order eigenvalues
ν_t	=	scalar eddy viscosity
$(\nu_{td})_i$	=	directional eddy-viscosity components
ν_{tt}, $(\nu_{tt})_{ij}$	=	tensorial eddy viscosity
Ω_{ij}	=	mean rotation tensor, $\frac{1}{2}(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i})$

I. Introduction

THE accurate prediction of the flow physics around bodies at high Reynolds number is a challenge in aerodynamics nowadays. In the context of turbulent flow modeling, recent advances like large eddy simulation (LES) and hybrid methods [detached eddy simulation (DES)] have considerably improved the physical relevance of the numerical simulation. However, the LES approach is still limited to the low-Reynolds-number range concerning wall flows. The unsteady Reynolds-averaged Navier–Stokes (URANS) approach remains a widespread and robust methodology for complex flow computation, especially in the near-wall region. Complex statistical models like second-order closure schemes [differential Reynolds stress modeling (DRSM)] improve the prediction of these properties and can provide an efficient simulation of turbulent stresses. From a computational point of view, the main drawbacks of such approaches are a higher cost, especially in unsteady 3-D flows and above all, numerical instabilities.

The linear eddy-viscosity models (EVM) use the Boussinesq approximation [1], which establishes a linear relation between the Reynolds stresses and the strain rate by means of a scalar eddy-viscosity concept, assuming a simple analogy with the relation standing for the molecular viscosity. The Boussinesq law can be written as follows under the incompressibility assumption:

$$-\frac{\overline{u_i u_j}}{k} + \frac{2}{3}\delta_{ij} = -a_{ij} = 2\frac{\nu_t}{k}S_{ij} \tag{1}$$

The scalar eddy viscosity is often expressed by means of the turbulence length and time scales as $\nu_t = C_\mu k^2/\varepsilon$.

A direct consequence of the Boussinesq approximation is that the principal directions of the two tensors $-a$ and S always remain collinear. This leads to an overproduction of turbulent kinetic energy [2], which occurs especially in flow regions upstream of the detachment, where the strain rate is high and the flow is laminar ([3,4], among others).

The class of nonlinear eddy-viscosity models (NLEVM) provides modified behavior laws that attempt to overcome the limitations mentioned. These laws are derived from a complete tensorial basis of the turbulent stresses [5,6] involving quadratic or cubic combinations of the strain and vorticity tensors. These laws are derived from algebraic forms of the turbulent stresses issued from the DRSM [7–9]. NLEVM use the scalar eddy-viscosity concept and lead to modified diffusion terms in the momentum equations, whose complexity increases with the higher-order terms involved in the tensorial basis of the constitutive law. The explicit algebraic stress models provide improved results for nonequilibrium flows [10], among others. Their predictive capacity of complex unsteady flows is still in progress (reported in [11,12]).

In the present paper, an alternative is suggested to derive a tensorial eddy-viscosity model sensitized for stress-strain misalignment and nonequilibrium turbulence. This approach, the Organized Eddy Simulation, follows previous studies that reached the prediction of massively detached unsteady turbulent flows around bodies by a reconsideration of the eddy-diffusion coefficient in two-equation models derived from DRSM [13–15]. This leads to a reduction of the eddy-diffusion coefficient which allowed a relevant prediction of the unsteady aerodynamic parameters, but this approach only used Boussinesq law as a first step. To provide an anisotropic analysis, which corresponds more to the flow physics, especially in the region close to flow detachment, it is worthwhile to extend the analysis by associating it with a directional criterion of nonequilibrium. The nonequilibrium can be illustrated by means of stress-strain misalignment [16], among other concepts, as well as by the ratio of the mean flow time scale over the turbulence time scale [8]. Both will be used in the present analysis. As discussed in this paper, one of the expected results will be a reduction of the eddy-diffusion coefficient, varying according to the nonequilibrium flow

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*UMR5502 CNRS-INP/UPS, Groupe EMT2, Allée du Professeur Camille Soula; bourguet@imft.fr.
†UMR5502 CNRS-INP/UPS, Groupe EMT2, Allée du Professeur Camille Soula; braza@imft.fr
‡UMR5502 CNRS-INP/UPS, Groupe EMT2, Allée du Professeur Camille Soula.

regions and the coherent flow structures, to reach an improved prediction of the turbulence production in respect of the flow physics. The underlying models are DRSM and two-equation models. The present developments are submitted to limitations coming from these same models: a scalar dissipation rate and a unique length/time scale, respectively.

To this end, a physical analysis of the stress-strain angles has been carried out on the basis of a detailed high-Reynolds particle image velocimetry (PIV) experiment concerning the incompressible flow past a circular cylinder at Reynolds number 140,000 in high blockage and aspect ratios [17]. The phase-averaged turbulence properties are considered, allowing distinction of the organized coherent physical process from the random turbulence. This experiment provides the quantification of the stress-strain principal direction misalignments in the near-wake region where turbulent equilibrium is not reached. Furthermore, anisotropic misalignment criteria are investigated and a tensorial definition of the eddy viscosity is put forward, leading to a new definition of the Reynolds stress constitutive law. Finally, transport equations derived from the Speziale et al. second-order closure scheme [18], are suggested to transport the misalignment criteria.

II. Investigation of Stress-Strain Misalignment via Phase-Averaged 3C-PIV Measurements in the Cylinder Wake

The near-periodic nature of the flow, due to the von Kármán vortices, allows the definition of a phase and the calculation of phase-averaged quantities. The procedure used is reported in [17]. In the present study, the median plan has been considered at half distance spanwise and located in the near-wake region. The phase-averaged decomposition is adopted for the whole analysis.

The angles between the principal directions of the strain rate and turbulence anisotropy tensors are quantified. The main coherent vortex regions are delimited by the Q criterion [19]. The first principal directions of each tensor are represented in Fig. 1a. In specific flow regions their misalignment becomes predominant. The largest misalignment is observed near the vortex center ($x_1/D = 1$, $x_2/D = 0.03$) in Fig. 1a. It decreases progressively and the best alignment is reached near the saddle point ($x_1/D = 1.18$, $x_2/D = 0.5$) and in free-shear flow regions.

In Fig. 1b the angle between the directions of v_1^a and v_1^S is represented for given ordinates (cf. bold lines in Fig. 1a). Solid and dashed-dotted curves ($x_2/D = -0.21$ and $x_2/D = -0.06$, respectively) confirm the misalignment peak near the vortex center (up to 50 deg around $x_1/D = 1$), whereas the dashed curve ($x_2/D = 0.39$) illustrates a quasi alignment near the saddle point and in free-flow regions (beyond $x_1/D = 1.5$).

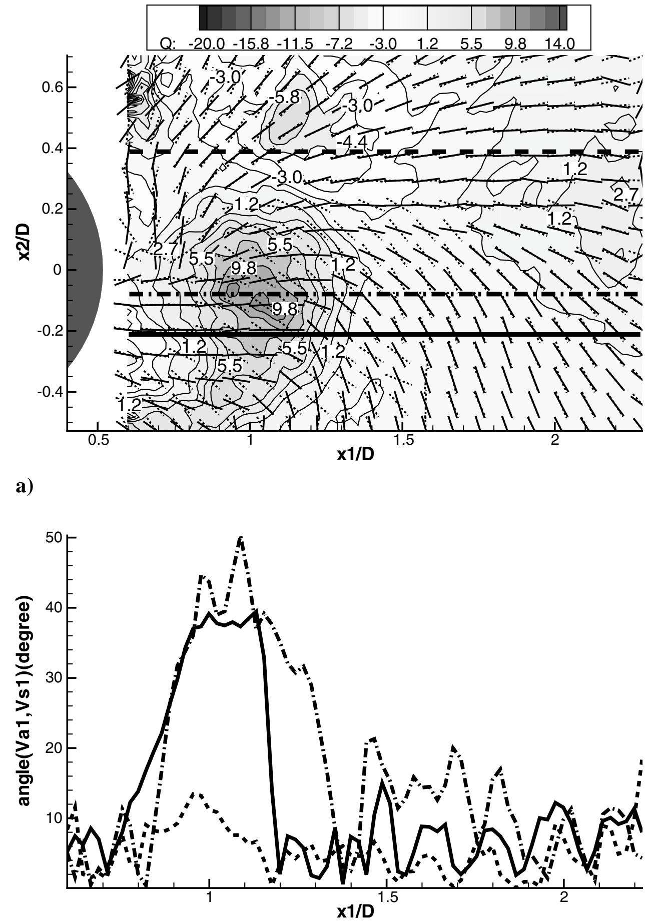

Fig. 1 a) $-a$ (dashed) and S (solid) first principal directions and Q criterion isocontours at phase $\varphi = 50$ deg. b) Angle variation between $-a$ and S first principal directions along the three lines in bold in a).

III. Towards an Anisotropic First-Order Eddy-Viscosity Model

A. Misalignment Estimation Criterion

To monitor the real misalignment between the three principal directions of the two tensors, a local evaluation of all eigenvalues and vectors is necessary. Unfortunately, such a computation implies an assumed knowledge about the two tensors, whereas the purpose of these angle evaluations is to include the misalignment effect in the Reynolds stress estimation. For this reason, a misalignment criterion has to be defined. This criterion must 1) give anisotropic information about tensor misalignment in each space direction, which means an evaluation of eigenvector correlations, and 2) be "advectable" through a specific transport equation that can be derived from DRSM as suggested in the preceding section.

Without any estimation of the eigenvectors of $-a$, the correlation rates between $-av_i^S$ and v_i^S provide sufficient information about the alignment between the principal directions of $-a$ and v_i^S, which leads to the following criterion definition:

$$C_i = -\frac{a_{jk}\left(v_i^S\right)_k\left(v_i^S\right)_j}{\|av_i^S\|} \quad \text{for } i = 1, 2, 3 \tag{2}$$

where $\|.\|$ is the Euclidian norm.

As can be shown in the present experiment, the criterion decreases in highly strained shear flow regions and especially near the vortex center, whereas it remains maximum when the two principal tensorial directions are aligned.

B. Tensorial Eddy-Viscosity Model

The three coefficients C_i provide anisotropic knowledge about the degree of linearity existing between Reynolds and the mean strain tensors. This to be so, even if the two tensors are aligned in one direction, a misalignment in another direction implies a nonlinearity between the tensors, it is relevant to consider the individual contribution of each element of a spectral decomposition that is applied to the strain-rate tensor. Moreover, according to other studies such as [8], for instance, $\eta = \frac{k\|S\|}{\varepsilon}$ mean flow/turbulent time-scale rate emphasizes the nonequilibrium turbulence regions. Whenever η is higher than 3.3 approximately, the nonequilibrium turbulence becomes predominant.

The following definition of an anisotropic eddy-diffusion coefficient can be suggested by an extension of the scalar C_μ definition, for $i = 1, 2, 3$:

$$C_{\mu i} = \frac{\left|a_{jk}\left(v_i^S\right)_k\left(v_i^S\right)_j\right|}{\eta} = |C_{Vi}|\frac{\varepsilon}{k} \tag{3}$$

where

$$C_{Vi} = -\frac{a_{jk}\left(v_i^S\right)_k\left(v_i^S\right)_j}{\|S\|} \tag{4}$$

Therefore, a positive directional eddy viscosity can be defined as follows:

$$(\nu_{td})_i = |C_{Vi}|k \tag{5}$$

Taking the previous analysis concerning the stress-strain misalignment into account, a consistent definition of the eddy-viscosity as a symmetric tensor is suggested:

$$(\nu_{tt})_{ij} = (\nu_{td})_k\left(v_k^S\right)_i\left(v_k^S\right)_j \tag{6}$$

Equation (6) leads to a weighted summation of S spectral decomposition:

$$S_{ik}(\nu_{tt})_{kj} = (\nu_{td})_l\lambda_l^S\left(v_l^S\right)_i\left(v_l^S\right)_j = (\nu_{td})_l(S_l)_{ij} \tag{7}$$

Furthermore, if the eddy viscosity is regarded as a 3×3 tensor, the linear EVM behavior law can be generalized as

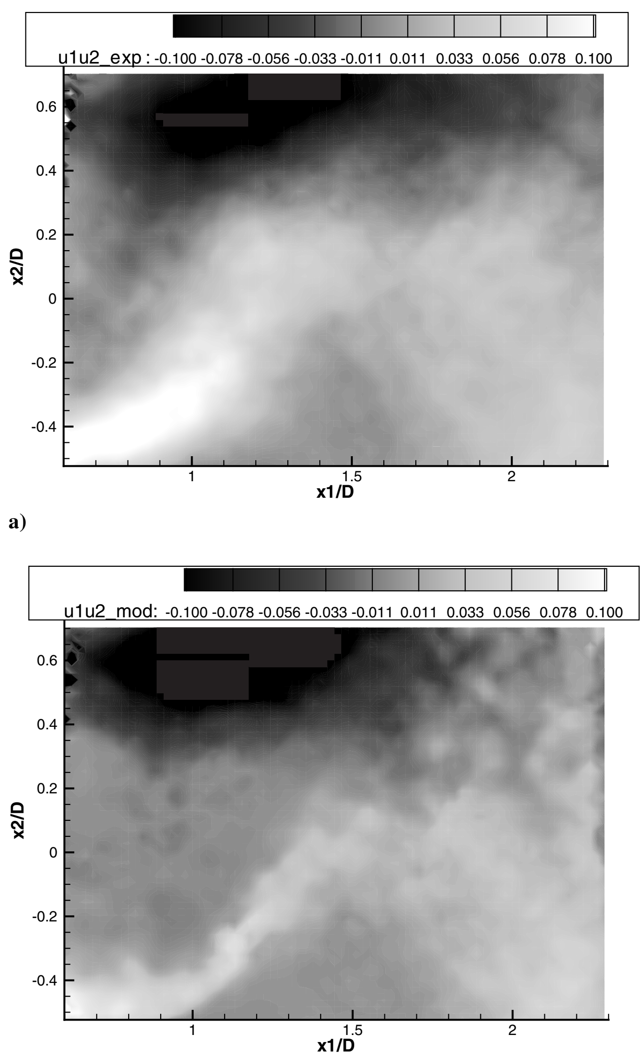

Fig. 2 Comparison between phase-averaged Reynolds stresses $\overline{u_iu_j}$ obtained a) directly from PIV experiment, and b) those evaluated via Eq. (8) and experimental strain tensor at phase $\varphi = 50$ deg.

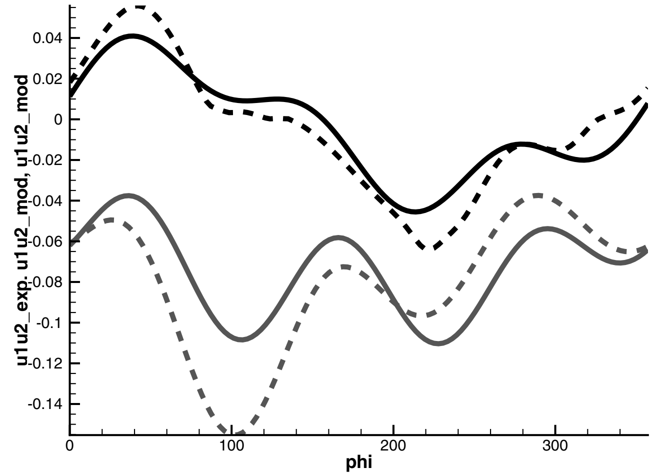

Fig. 3 Comparison between measured phase-averaged Reynolds stresses (solid) and modeled quantities (dashed) as a function of the phase angle, within a period at location points ($x_1/D = 0.73$, $x_2/D = 0.55$) (gray) and ($x_1/D = 1.82, x_2/D = 0.08$) (black).

$$-\overline{u_iu_j} + \frac{2}{3}k\delta_{ij} = 2S_{ik}(\nu_{tt})_{kj} - \frac{2}{3}R\delta_{ij} \tag{8}$$

where $R = (\nu_{td})_i\lambda_i^S$ is the trace of $S_{ik}(\nu_{tt})_{kj}$. From Eq. (7), the symmetry property of the turbulence anisotropy tensor is ensured. $-\frac{2}{3}R\delta_{ij}$ term provides Eq. (8) validity when summing $\overline{u_iu_i}$. Equation (8) leads to the following generalization of averaged Navier–Stokes momentum equations:

$$\frac{DU_i}{Dt} = \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + (\nu_{tt})_{kj}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\right] - \frac{\partial}{\partial x_i}\left[P + \frac{2}{3}\left(k + R\right)\right] \tag{9}$$

The EVM suggested in (8) is no longer linear because ν_{tt} depends on local directional flow characteristics and on η. The tensorial definition enables a selective reduction of the effect of one (or more) elements of the strain-rate spectral decomposition with respect to the corresponding physical alignment (or misalignment) between the associated principal directions. Moreover, if a perfect alignment is observed in an equilibrium and isotropic strain region, that is to say $|\lambda_1^S| \approx |\lambda_2^S| \approx |\lambda_3^S|$, then $(\nu_{td})_i = \nu_t$ for all i, $S_{ik}(\nu_{tt})_{kj} = \nu_t\lambda_l^S(v_l^S)_i(v_l^S)_j = \nu_t S_{ij}$, and $R = \nu_t\lambda_i^S = 0$, which means that the tensorial expression degenerates into a classical Boussinesq-like scalar model.

A comparison between Reynolds stresses evaluated from the PIV experiment and from modeling via (8) and measured stress tensor is shown in Fig. 2. Despite slight differences in shear flow regions, the modeled quantities present a good match with the experiment for both normal and shear Reynolds stresses. This is verified over the whole period of the vortex shedding (Fig. 3). The new model provides quite a good comparison with the experimental results. It can be remembered that considerable discrepancies occur through Boussinesq-based two-equation models, especially for the shear stress prediction in the near-wall and near-wake regions, as mentioned in [11,12]. Therefore, the present model gives satisfactory results for a physical experiment that allows accessing detailed fields of turbulence quantities relevant to the flow physics. This is an intermediate phase of the study related to the present model that could be implemented in computational fluid dynamics (CFD) codes in comparison with other behavior laws issued from nonlinear modeling.

C. Transport Equations for the Misalignment Criteria

To transport these coefficients as new variables of the physical system, three advection equations can be derived from the DRSM model by Speziale, Sarkar, and Gatski (SSG model) [18]. For $q = 1$, 2, 3,

$$\frac{DC_{Vq}}{Dt} = -\frac{1}{\|S\|}\left((V_q)_{ij}\frac{Da_{ij}}{Dt} + a_{ij}\frac{D(V_q)_{ij}}{Dt} + C_{Vq}\frac{D\|S\|}{Dt}\right) \quad (10)$$

where $(V_q)_{ij} = (v_q^S)_i(v_q^S)_j$.

By using the SSG model for the pressure/strain-rate correlation in a similar way as [20] for a nondirectional misalignment, the expression (10) leads to the following equations:

$$\frac{DC_{Vq}}{Dt} = \left(\frac{4}{3} + c_3^* II_a^{\frac{1}{2}} - c_3\right)\frac{(V_q)_{ij}S_{ij}}{\|S\|} + (2-2c_4)\frac{(V_q)_{ij}a_{ik}S_{jk}}{\|S\|} + (2-2c_5)\frac{(V_q)_{ij}a_{ik}\Omega_{jk}}{\|S\|} + (1-c_1)\frac{\varepsilon}{k}C_{Vq} + \left(1+c_1^*\right)C_{Vq}a_{ij}S_{ij} - \frac{c_2}{\eta}(V_q)_{ij}a_{ik}a_{kj} + \frac{c_2 II_a}{3\eta} + \frac{2(c_4-1)}{3}\frac{a_{ij}S_{ij}}{\|S\|} - \frac{1}{\|S\|}\left(a_{ij}\frac{D(V_q)_{ij}}{Dt} + C_{Vq}\frac{D\|S\|}{Dt}\right) + D^{C_{Vq}} \quad (11)$$

where $D^{C_{Vq}}$, the diffusion term, can be approximated by

$$D^{C_{Vq}} = \frac{\partial}{\partial x_i}\left[\left(\nu + \frac{(\nu_{tt})_{ij}}{\sigma_{C_{Vq}}}\right)\frac{\partial C_{Vq}}{\partial x_j}\right] \quad (12)$$

$II_a = a_{ij}a_{ij}$, and the seven constants c_i and c_i^* are those determined by Speziale et al. [18].

$D^{C_{Vq}}$ is a suggestion for modeling the diffusive term of C_{Vq} coefficients combining viscous and turbulent diffusion contributions. Assuming a similarity with the diffusion term of k transport equation, $\sigma_{C_{Vq}}$ coefficient can be set, firstly, to the value of 1.

IV. Conclusions

The present study quantifies the existence of a strong misalignment between the phase-averaged turbulence stresses and the strain-rate tensor in the coherent vortices and in the highly sheared regions downstream of the separation. A transportable misalignment angle criterion is suggested in a general form, consistent in three dimensions. This yields an anisotropic tensorial eddy-viscosity concept sensitized in respect of the nonequilibrium turbulence. A good match between the modeled turbulence stresses and the experimental results under the phase-averaged decomposition is reached. This is an ongoing study which aims at investigating anisotropic nonequilibrium criteria for turbulence modeling of highly detached unsteady flows.

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R. So
Associate Editor